

# Direct Integration Preconditioning For Solving Optimal Control Problems

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## Abstract

In this paper new explicit expressions for both kinds of Comtet numbers and some interesting special cases are derived. Moreover, we define and study the generalized multiparameter non-central Stirling numbers and generalized Comtet numbers via differential operators. Furthermore, recurrence relations and new explicit formulas for those numbers are obtained. Finally some interesting special cases, new combinatorial identities and a connection between these numbers and some interesting polynomials are deduced. (C) 2011 Elsevier Ltd. All rights reserved.

**KeyWords:** Differential operators; Generalized multiparameter non-central Stirling numbers; Generalized Comtet numbers; Generalized Truesdell polynomials; Generalized Bell polynomials; Generalized Dobinski formula

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## References:

- [1] B.S. El-Desouky, Multiparameter non-central Stirling numbers, *Fibonacci Quart.* 32 (3) (1994) 218–220.
- [2] L. Comtet, Nombres de Stirling généraux et fonctions symétriques, *C. R. Acad. Sci. Paris. (Ser. A)* 270 (1972) 747–750.
- [3] T. Bickel, The group of generalized Stirling numbers, *Adv. Appl. Math.* 26 (2001) 1–22.
- [4] R.B. Corcino, Some theorems on generalized Stirling numbers, *Ars Combin.* 60 (2001) 273–281.
- [5] C.G. Wagner, Generalized stirling and Lah numbers, *Discrete Math.* 160 (1996) 199–218.
- [6] J. Konvalina, Generalized binomial coefficients and the subset-subspace problem, *Adv. Appl. Math.* 21 (1998) 228–240.
- [7] C.G. Wagner, Partition statistics and q-Bell numbers ( $q = -1$ ), *J. Integer Seq.* 8 (2005) Article 04.1.1.
- [8] M. Petkovsek, H.S. Wilf, D. Zeilberger, *A=B*, A.K. Peters, Wellesley, MA, 1997.
- [9] H.S. Wilf, *Generating Functionology*, Academic Press Inc., New York, 1994.
- [10] B.S. El-Desouky, N.P. Cakić, T. Mansour, Modified approach to generalized Stirling numbers via differential operators, *Appl. Math. Lett.* 23 (2010) 110–115.
- [11] R.L. Graham, D.E. Knuth, O. Patashnik, *Concrete Mathematics*, second ed., Reading, MA: Addison-Wesley Publishing Company, 1994.
- [12] H.W. Gould, Combinatorial numbers and associated identities, table 1: stirling numbers, Unpublished Manuscript,  
<http://www.math.wvu.edu/gould/Vol.V>.

- [<sup>13</sup>] G. Dattoli, M.X. He, P.E. Ricci, Eigenfunctions of Laguerre-type operators and generalized in evolution problems, *Math. Comput. Modelling* **42** (11–12) (2000) 1263–1268.
- [<sup>14</sup>] O.V. Viskov, H.M. Srivastava, New approaches to certain identities involving differential operators, *J. Math. Anal. Appl.* **186** (1994) 1–10.
- [<sup>15</sup>] P. Blasiak, Combinatorics of Boson Normal Ordering and Some Applications, Ph.D. Thesis, University of Paris, 2000.
- [<sup>16</sup>] P. Blasiak, K.A. Penson, A.I. Solomon, The normal ordering general boson problem, *Phys. Lett. A* **309** (2003) 198–200.
- [<sup>17</sup>] L. Comtet, Advanced Combinatorics: The Art of Finite and Infinite Expansions, D. Reidel Publishing Company, Dordrecht, Holland, 1974.
- [<sup>18</sup>] T. Bier, P.S. Padmanabhan, Some formulas for generalized Stirling numbers, *Ars Combin.* **LXXVI** (2000).
- [<sup>19</sup>] M. Koutras, Non-central Stirling numbers and some applications, *Discrete Math.* **42** (1982) 73–89.
- [<sup>20</sup>] Wenchang Chu, Chuanam Wei, Set partitions with restrictions, *Discrete Math.* **308** (2008) 3163–3168.
- [<sup>21</sup>] P.N. Shrivastava, On the polynomials of truesdel type, *Publ. Inst. Mat. (N.S.)* **9** (22) (1969) 43–46.
- [<sup>22</sup>] R.P. Singh, On generalized truesdel polynomials, *Riv. Mat. Univ. Parma* **7** (8) (1977) 345–352.

### **Generalized higher order Stirling numbers**

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### **Abstract**

In this paper, direct integration preconditioning is proposed to solve optimal control problems governed by ordinary differential equations. Legendre approximations are used to reduce the problem to a constrained optimization problem. Error estimation for the Legendre approximations is derived and a technique that gives an optimal approximation of the problems is introduced. Numerical results are included to confirm the efficiency and accuracy of the method.

**KeyWords:** Spectral methods; approximation by Legendre polynomials; optimal control problems

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### **References:**

1- Author(s): Canuto, C; Hussaini, MY; Quarteroni, A; et al. Source: Spectral Methods in Fluid Dynamics Published: 1988 Publisher: Springer-Verlag, Berlin

2- Title: The ultraspherical coefficients of the moments of a general-order derivative of an infinitely differentiable function Author(s): Doha, EH Source: JOURNAL OF

- COMPUTATIONAL AND APPLIED MATHEMATICS Volume: ۸۹ Issue: ۱  
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- ۳- Title: A CHEBYSHEV-APPROXIMATION FOR SOLVING OPTIMAL-CONTROL PROBLEMS Author(s): ELGINDY, TM; ELHAWARY, HM; SALIM, MS; et al.Source: COMPUTERS & MATHEMATICS WITH APPLICATIONS Volume: ۲۹ Issue: ۶ Pages: ۳۰-۴۰ DOI: ۱۰.۱۰۱۶/۰۸۹۸-۱۲۲۱(۹۰)۰-۰-۰-J Abstract Number: C۱۹۹۰-۰۳-۱۲۳-۰-۷۰ Published: MAR ۱۹۹۰
- ۴- Title: [not available] Author(s): ELGINDY TM Source: J I MATH COMPUT SCI Volume: ۳ Pages: ۸۰ Published: ۱۹۹۰ .
- ۵- Title: [not available] Author(s): ELHAWARY HM Source: INT J COMPUT IN PRESS Volume: ۲۰ Published: ۲۰۰۰ .
- ۶- Title: [not available] Author(s): ELKADY M Source: COMMUNICATIONS UNPUB Published: ۲۰۰۹ .
- ۷- Title: [not available] Author(s): ELNAGAR GM Source: J COMPUT APPL MATH Volume: ۹۷ Pages: ۱۹ Published: ۱۹۹۷ .
- ۸- Title: [not available] Author(s): Falb, P.L.; Jong, J.L. Source: Some Successive Approximation Methods on Control and Oscillation Theory Published: ۱۹۶۹ .
- ۹- Title: [not available] Author(s): Gottlieb, D.; Orszag, S. A. Source: Numerical Analysis of Spectral Methods: Theory and Applications Volume: ۲۶ Published: ۱۹۷۷ Publisher: SIAM Publications, Philadelphia, PA
- ۱۰- Title: [not available] Author(s): MARTIN G Source: J COMPUT APPL MATH Volume: ۱۱۴ Pages: ۱۰۳ Published: ۲۰۰۰ .
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### **Modified approach to generalized Stirling numbers via differential operators**

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**KeyWords:** Stirling numbers; Generalized Stirling numbers; Combinatorial identities; Normal ordering; Boson operators

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**References:**

- [1] L. Toscano, Numeri di Stirling generalizzati, operatori differenziali e polinomi ipergeometrici, *Pontificia Academia Scientiarum Commentationes* 4 (1939) 721–757.
- [2] L. Toscano, Sulla iterazione dell' operatore  $xD$ , *Rend. Mat. Appl.* 8 (1949) 227–300.
- [3] L. Carlitz, M.S. Klamkin, Stirling operators, *Collect. Math.* XXV (2) (1974) 186–211.
- [4] J. Katriel, Combinatorial aspects of Boson algebra, *Lett. Nuovo Cimento* 1 (1974) 560–567.
- [5] N.J.A. Sloane, The on-line encyclopedia of integer sequences, 2008.
- [6] L. Comtet, Nombres de Stirling généraux et fonctions symétriques, *C. R. Acad. Sci. Paris* 270 (1972) 747–750.
- [7] Ch.A. Charalambides, J. Singh, A review of the Stirling numbers, their generalizations and statistical applications, *Comm. Statist. Theory Methods* 17 (8) (1988) 2033–2090.
- [8] K.A. Penson, P. Blasiak, G. Duchamp, A. Horzela, A.I. Solomon, Hierarchical Dobinski-type relations via substitution and the moment problem, *J. Phys. A: Math. Gen.* 37 (2004) 3470–3487.
- [9] T. Mansour, M. Schork, On the normal ordering of multi-mode Boson operators, *Russ. J. Math. Phys.* 10 (1) (2004) 50–61.
- [10] P. Blasiak, K.A. Penson, A.I. Solomon, The normal ordering general Boson problem, *Phys. Lett. A* 309 (2003) 198–200.
- [11] P. Blasiak, Combinatorics of Boson normal ordering and some applications, Ph.D. Thesis, University of Paris VI and Polish Academy of Sciences, Krakow, Poland, 2000. arXiv:quant-ph/0007207.
- [12] H.M. Srivastava, Vijay Gupta, Rate of convergence for the Bézier variant of the Bleimann\_Butzer\_Hahn operators, *Appl. Math. Lett.* 18 (8) (2005) 849–857.
- [13] T. Mansour, M. Schork, S. Severini, A generalization of Boson normal ordering, *Phys. Lett. A* 364 (2007) 214–220.
- [14] L. Carlitz, On arrays of numbers, *Amer. J. Math.* 54 (1932) 739–752.
- [15] N.P. Cakić, On some combinatorial identities, *Univ. Beograd. Publ. Elektrotehn. Fak. Ser. Mat. Fiz.* 678–710 (1980) 91–94.
- [16] G.V. Milovanović, N.P. Cakić, Explicit formulas for numbers of Carlitz and Toscano, *Facta Univ. Ser. Math. Inform.* 9 (1994) 1–10.
- [17] A.M. Chak, A class of polynomials and a generalization of Stirling numbers, *Duke Math. J.* 23 (1957) 45–50.

- [18] W. Lang, On generalizations of the Stirling number triangles, J. Integer Seq. 7 (2) (2004) Article 04.2.4.
- [19] B.S. El-Desouky, The multiparameter non-central Stirling numbers, Fibonacci Quart. 32 (1994) 218–220.
- [20] L.C. Hsu, P.J-S. Shiue, A unified approach to generalized Stirling numbers, Adv. in Appl. Math. 20 (1998) 366–384.
- [21] A. Regev, Y. Roichman, Statistics on wreath products and generalized binomial-Stirling numbers, Israel J. Math. 101 (2007) 189–221.
- [22] M. Petkovsek, Toma Pisansk, Combinatorial interpretation of unsigned Stirling and Lah numbers, preprint. [http://www.fmf.uni-lj.si/\\_petkovsek/stirlinglah.ps](http://www.fmf.uni-lj.si/_petkovsek/stirlinglah.ps).