

## Thermomagnetic viscoelastic responses in a functionally graded hollow structure

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**Abstract:**

This paper presents an analytical solution for the interaction of electric potentials, electric displacements, elastic deformations, and thermoelasticity, and describes electromagnetoelastic responses and perturbation of the magnetic field vector in hollow structures (cylinder or sphere), subjected to mechanical load and electric potential. The material properties, thermal expansion coefficient and magnetic permeability of the structure are assumed to be graded in the radial direction by a power law distribution. In the present model we consider the solution for the case of a hollow structure made of viscoelastic isotropic material, reinforced by elastic isotropic fibers, this material is considered as structurally anisotropic material. The exact solutions for stresses and perturbations of the magnetic field vector in FGM hollow structures are determined using the infinitesimal theory of magnetothermoelasticity, and then the hollow structure model with viscoelastic material is solved using the correspondence principle and Illyushin's approximation method. Finally, numerical results are carried out and discussed.

**KeyWords:** Functionally graded material; Viscoelasticity; Perturbation of magnetic field vector; Magnetoelasticity

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**References:**

1. Reddy, J.N.: Analysis of functionally graded plates. *Int. J. Numer. Meth. Eng.* **47**, 663–684 (2000) Suresh, S., Mortensen, A.: *Fundamentals of Functionally Graded Materials*, IOM Communications Limited, London (1998)
2. Koizumi, M.: FGM activities in Japan. *Composites* **28**(1–2), 1–4 (1997)
3. Tanigawa, Y., Matsumoto, M., Akai, T.: Optimization of material composition to minimize thermal stresses in nonhomogeneous plate subjected to unsteady heat supply. *JPN Soc. Mech. Engrs. Int. J. Ser. A* **40**(1), 84–93 (1997)
4. Zimmerman, R.W., Lutz, M.P.: Thermal stresses and thermal expansion in a uniformly heated functionally graded cylinder. *J. Thermal Stresses* **22**, 177–188 (1999) Sankar, B.V.: An elasticity solution for functionally graded beams. *Compos. Sci. Tech.* **61**, 689–696 (2001) Zenkour, A.M.: Benchmark trigonometric and 3-D elasticity solutions for an exponentially graded thick rectangular plate. *Arch. Appl. Mech.* **77**, 197–214 (2007) Zenkour, A.M., Elsibai, K.A., Mashat, D.S.: Elastic and viscoelastic solutions to rotating functionally graded hollow and solid cylinders. *App. Math. Mech. Engl. Ed.* **29**(12), 1601–1616 (2008) Arciniega, R.A., Reddy, J.N.: Large deformation analysis of functionally graded shells. *Int. J. Solids Struct.* **44**, 2036–2052 (2007) Kadoli, R., Akhtar, K., Ganesan, N.: Static analysis of functionally graded beams using higher order shear deformation theory. *App. Math. Modell.* **32**, 2509–2525 (2008) Praveen, G.N., Reddy, J.N.: Nonlinear transient thermoelastic analysis of functionally graded ceramic-metal plates. *Int. J. Solids Struct.* **35**, 4457–4476 (1998) Reddy, J.N., Chin, C.D.:

- Thermomechanical analysis of functionally graded cylinders and plates. *J. Thermal Stresses* **21**, 593–626 (1998) Reddy, J.N., Cheng, Z.Q.: Three-dimensional thermomechanical deformations of functionally graded rectangular plates. *Eur. J. Mech. A Solids* **20**, 841–855 (2001) Zenkour, A.M.: A comprehensive analysis of functionally graded sandwich plates, Part 1: deflection and stresses. *Int. J. Solids Struct.* **42**, 5224–5242 (2005)
5. Zenkour, A.M.: A comprehensive analysis of functionally graded sandwich plates, Part 2: buckling and free vibration. *Int. J. Solids Struct.* **42**, 5243–5258 (2005)
  6. Zenkour, A.M.: Generalized shear deformation theory for bending analysis of functionally graded plates. *App. Math. Model.* **30**, 67–84 (2006)
  7. Zenkour, A.M., Alghamdi, N.A.: Thermoelastic bending analysis of functionally graded sandwich plates. *J. Mater. Sci.* **43**, 2574–2589 (2008)
  8. Chakraborty, A., Gopalakrishnan, S., Reddy, J.N.: A new beam finite element for the analysis of functionally graded materials. *Int. J. Mech. Sci.* **45**, 519–539 (2003)
  9. Nadeau, J.C., Ferrari, M.: Microstructural optimization of a functionally graded transversely isotropic layer. *Mech. Mater.* **31**, 637–651 (1999)
  10. Naki, T., Murat, O.: Exact solutions for stresses in functionally graded pressure vessels. *Composite B* **32**, 683–686 (2001)
  11. Dai, H.L., Fu, Y.M.: Magneto-thermoelastic interactions in hollow structures of functionally graded material subjected to mechanical loads. *J. Pressure Vessels and Piping* **84**, 132–138 (2007)
  12. Ghosh, M.K., Kanoria, M.: Analysis of thermoelastic response in a functionally graded spherically isotropic hollow sphere based on Green-Lindsay theory. *Acta Mech.* **207**, 51–67 (2009)
  13. Li, X.Y., Ding, H.J., Chen, W.Q.: Axisymmetric elasticity solutions for a uniformly loaded annular plate of transversely isotropic functionally graded materials. *Acta Mech.* **196**, 139–159 (2008)
  14. Ueda, S.: A cracked functionally graded piezoelectric material strip under transient thermal loading. *Acta Mech.* **199**, 53–70 (2008)
  15. Allam, M.N.M., Badr, R.E., Tantawy, R.: Stresses of a rotating circular disk of variable thickness carrying a current and bearing a coaxial viscoelastic coating. *App. Math. Model.* **32**, 1643–1656 (2008)
  16. Allam, M.N.M., Pobedrya, B.E.: On the solution of quasi-static problem in anisotropic viscoelasticity. *ISVAcad. Nauk. Ar. SSR Mech.* **31**, 19–27 (1976) (in Russian)
  17. Allam, M.N.M., Zenkour, A.M., El-Mekawy, H.F.: Stress concentrations in a viscoelastic composite plate weakened by a triangular hole. *Compos. Struct.* **79**, 1–11 (2007)
  18. Il'yushin, A.A., Pobedrya, B.E.: *Foundation of Mathematical Theory of Thermo-Viscoelasticity*. Nauka, Moscow (1970) (in Russian)
  19. Allam, M.N.M., Appleby, P.G.: On the stress concentrations around a circular hole in a fiber reinforced viscoelastic plate. *Res. Mech.* **19**, 113–126 (1986)
  20. Allam, M.N.M., Zenkour, A.M.: Bending response of a fiber-reinforced viscoelastic arched bridge model. *Appl. Math. Model.* **27**, 233–248 (2003)
  21. Zenkour, A.M.: Buckling of fiber-reinforced viscoelastic composite plates using various plate theories. *J. Eng. Math.* **50**, 75–93 (2004)
  22. Ezzat, M.A.: Generation of generalized thermomagnetoelastic waves by thermal shock in a perfectly conducting half-space. *J. Thermal Stresses* **20**, 633–917 (1997)

23. Kraus, J.D.: Electromagnetic, McGraw Hill, Inc., USA (1984)
24. Dai, H.L., Wang, X.: Dynamic responses of piezoelectric hollow cylinders in an axial magnetic field. *Int. J. Solids Struct.* **41**, 5231–5246 (2004)
25. Allam, M.N.M., Badr, R.E., Tantawy, R.: Stresses of a rotating circular disk of variable thickness carrying a current and bearing a coaxial viscoelastic coating. *App. Math. Model* **32**, 1643–1656 (2008)
26. Allam, M.N.M., Pobedrya, B.E.: On the solution of quasi-static problem in anisotropic viscoelasticity. *ISVAcad. Nauk. Ar. SSR Mech.* **31**, 19–27 (1976) (in Russian)
27. Allam, M.N.M., Zenkour, A.M., El-Mekawy, H.F.: Stress concentrations in a viscoelastic composite plate weakened by a triangular hole. *Compos. Struct.* **79**, 1–11 (2007)
28. Illyushin, A.A., Pobedrya, B.E.: Foundation of Mathematical Theory of Thermo Viscoelasticity. Nauka, Moscow (1970) (in Russian)
29. Allam, M.N.M., Appleby, P.G.: On the stress concentrations around a circular hole in a fiber reinforced viscoelastic plate. *Res. Mech.* **19**, 113–126 (1986)
30. Allam, M.N.M., Zenkour, A.M.: Bending response of a fiberreinforced viscoelastic arched bridge model. *Appl. Math. Model* **27**, 233–248 (2003)
31. Zenkour, A.M.: Buckling of fiber-reinforced viscoelastic composite plates using various plate theories. *J. Eng. Math.* **50**, 75–93 (2004)
32. Ezzat, MA.: Generation of generalized thermomagnetoelastic waves by thermal shock in a perfectly conducting half-space. *J. Thermal Stresses* **20**, 633–917 (1997)
33. Kraus, J.D.: Electromagnetic, McGraw Hill, Inc., USA (1984)
34. Dai, H.L., Wang, X.: Dynamic responses of piezoelectric hollow cylinders in an axial magnetic field. *Int. J. Solids Struct.* **41**, 5231–5246 (2004)

## On the simple and mixed first-order theories for plates resting on elastic foundations

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Abstract:

This article investigates the bending response of an orthotropic rectangular plate resting on two-parameter elastic foundations. Analytical solutions for deflection and stresses are developed by means of the simple and mixed first-order shear deformation plate theories. The present mixed plate theory accounts for variable transverse shear stress distributions through the thickness and does not require a shear correction factor. The governing equations that include the interaction between the plate and the foundations are obtained. Numerical results are presented to demonstrate the behavior of the system. The results are compared with those obtained in the literature using three-dimensional elasticity theory or higher-order shear deformation plate theory to check the accuracy of the simple and mixed first-order shear deformation theories.

**KeyWords:** DIFFERENTIAL QUADRATURE METHOD; FINITE-ELEMENT FORMULATION; VARIATIONAL FORMULA; LAMINATED PLATES; FREE-VIBRATION; DYNAMIC-RESPONSE; THICK PLATES; BEAMS

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**References:**

1. Akavci S.S., Yerli H.R., Dogan A.: The first order shear deformation theory for symmetrically laminated composite plates on elastic foundation. Arab. J. Sci. Eng. **32**, 341–348 (2007)
2. Jaiswal O.R., Iyengar R.N.: Dynamic response of a beam on elastic foundation of finite depth under a moving force. Acta Mech. **96**, 67–83 (1993)
3. Chudinovich I., Constanda C.: Integral representations of the solutions for a bending plate on an elastic foundation. Acta Mech. **139**, 33–42 (2000)
4. Tsiatas G.C.: Nonlinear analysis of non-uniform beams on nonlinear elastic foundation. Acta Mech. **209**, 141–152 (2010)
5. Dumir P.C.: Nonlinear dynamic response of isotropic thin rectangular plates on elastic foundations. Acta Mech. **71**, 233–244 (2003)
6. Chien R.D., Chen C.S.: Nonlinear vibration of laminated plates on an elastic foundation. Thin-Walled Struct. **44**, 852–860 (2006)
7. Celep Z., Güler K.: Axisymmetric forced vibrations of an elastic free circular plate on a tensionless two parameter foundation. J. Sound Vib. **301**, 495–509 (2007)
8. Liew K.M., Han J.B., Xiao Z.M., Du H.: Differential quadrature method for Mindlin plates on Winkler foundations. Int. J. Mech. Sci. **38**, 405–421 (1996)

## Bending of a fiber-reinforced viscoelastic composite plate resting on elastic foundations

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**Abstract:**

Composite structures on an elastic foundation are being widely used in engineering applications. Bending response of inhomogeneous viscoelastic plate as a composite structure on a two-parameter (Pasternak's type) elastic foundation is investigated. The formulations are based on sinusoidal shear deformation plate theory. Trigonometric terms are used in the present theory for the displacements in addition to the initial terms of a power series through the thickness. The transverse shear correction factors are not needed because a correct representation of the transverse shear strain is given. The interaction between the plate and the foundation is included in the formulation with a two-parameter Pasternak's model. The effective moduli and Illyushin's approximation methods are used to derive the viscoelastic solution. The effects played by foundation stiffness, plate aspect ratio, and other parameters are presented.

**KeyWords:** Bending response; Viscoelastic composite plate; Elastic foundation; Sinusoidal shear deformation plate theory.

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**References:**

1. Borisovich A., Dymkowska J., Szymczak C.: Buckling and postcritical behaviour of the elastic infinite plate strip resting on linear elastic foundation. *J. Math. Anal. Appl.* **307**, 480–495 (2005)
2. Lal R.: Dhanpati: Transverse vibrations of non-homogeneous orthotropic rectangular plates of variable thickness: a spline technique. *J. Sound Vib.* **306**, 203–214 (2007)
3. Chucheepsakul S., Chinnaboon B.: Plates on two-parameter elastic foundations with nonlinear boundary conditions by the boundary element method. *Comput. Struct.* **81**, 2739–2748 (2003)
4. Shen H.S.: Nonlinear bending of shear deformation laminated plates under transverse and in-plane loads and resting on elastic foundations. *Compos. Struct.* **50**, 131–142 (2000)
5. Shen H.S.: Large deflection of composite laminated plates under transverse and in-plane loads and resting on elastic foundations. *Compos. Struct.* **45**, 115–123 (1999)
6. Singh B.N., Lal A., Kumar R.: Post buckling response of laminated composite plate on elastic foundation with random system properties. *Commun. Nonlinear Sci. Numer. Simul.* **14**, 284–300 (2009)

# Bending analysis of FG viscoelastic sandwich beams with elastic cores resting on Pasternak's elastic foundations

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## **Abstract:**

The investigation of bending response of a simply supported functionally graded (FG) viscoelastic sandwich beam with elastic core resting on Pasternak's elastic foundations is presented. The faces of the sandwich beam are made of FG viscoelastic material while the core is still elastic. Material properties are graded from the elastic interfaces through the viscoelastic faces of the beam. The elastic parameters of the faces are considered to be varying according to a power-law distribution in terms of the volume fraction of the constituent. The interaction between the beam and the foundations is included in the formulation. Numerical results for deflections and stresses obtained using the refined sinusoidal shear deformation beam theory are compared with those obtained using the simple sinusoidal shear deformation beam theory, higher- and first-order shear deformation beam theories. The effects due to material distribution, span-to-thickness ratio, foundation stiffness and time parameter on the deflection and stresses are investigated.

**Keywords:** FUNCTIONALLY GRADED BEAMS; FINITE-ELEMENT FORMULATION; SHEAR DEFORMATION-THEORY; COMPREHENSIVE ANALYSIS; COMPOSITE BEAMS; FREE-VIBRATION; PLATES; INTERFACES; CYLINDERS; BEHAVIOR

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## **References:**

1. Shi Y., Sol H., Hua H.: Material parameter identification of sandwich beams by an inverse method. *J. Sound Vib.* **290**, 1234–1255 (2006)
2. Yim J.H., Cho S.Y., Seo Y.J., Jang B.Z.: A study on material damping of 0° laminated composite sandwich cantilever beams with a viscoelastic layer. *Compos. Struct.* **60**, 367–374 (2003)
3. Barbosa F.S., Farage M.C.R.: A finite element model for sandwich viscoelastic beams: Experimental and numerical assessment. *J. Sound Vib.* **317**, 91–111 (2008)
4. Bekuit J.-J.R.B., Oguamanam D.C.D., Damisa O.: A quasi-2D finite element formulation for the analysis of sandwich beams. *Fin. Elem. Anal. Des.* **43**, 1099–1107 (2007)
5. Nayfeh S.A.: Damping of flexural vibration in the plane of lamination of elastic-viscoelastic sandwich beams. *J. Sound Vib.* **276**, 689–711 (2004)
6. Lin C.Y., Chen L.W.: Dynamic stability of rotating composite beams with a viscoelastic core. *Compos. Struct.* **58**, 185–194 (2002)

7. Yan W., Chen W.Q., Wang B.S.: On time-dependent behavior of cross-ply laminated strips with viscoelastic interfaces. *Appl. Math. Model.* **31**, 381–391 (2007)
8. Barkanov E., Rikards R., Holste C., Täger O.: Transient response of sandwich viscoelastic beams, plates, and shells under impulse loading. *Mech. Compos. Mater.* **36**, 215–222 (2000)
9. Galucio A.C., Deü J.-F., Ohayon R.: Finite element formulation of viscoelastic sandwich beams using fractional derivative operators. *Comput. Mech.* **33**, 282–291 (2004)
10. Yen J.Y., Chen L.W., Wang C.C.: Dynamic stability of a sandwich beam with a constrained layer and electrorheological fluid core. *Compos. Struct.* **84**, 209–219 (2008)
11. Yan W., Ying J., Chen W.Q.: Response of laminated adaptive composite beams with viscoelastic interfaces. *Compos. Struct.* **74**, 70–79 (2006)
12. Teng T.L., Hu N.K.: Analysis of damping characteristics for viscoelastic laminated beams. *Comput. Methods Appl. Mech. Engrg.* **190**, 3881–3892 (2001)
13. Beldica C.E., Hilton H.H.: Nonlinear viscoelastic beam bending with piezoelectric control—analytical and computational simulations. *Compos. Struct.* **51**, 195–203 (2001)
14. Zenkour A.M.: Benchmark trigonometric and 3-D elasticity solutions for an exponentially graded thick rectangular plate. *Arch. Appl. Mech.* **77**, 197–214 (2007)
15. Zenkour A.M., Elsibai K.A., Mashat D.S.: Elastic and viscoelastic solutions to rotating functionally graded hollow and solid cylinders. *Appl. Math. Mech. Engl. Ed.* **29**(12), 1601–1616 (2008)
16. Sankar B.V.: An elasticity solution for functionally graded beams. *Compos. Sci. Tech.* **61**, 689–696 (2001)
17. Zenkour A.M.: A comprehensive analysis of functionally graded sandwich plates: Part 1-deflection and stresses. *Int. J. Solids Struct.* **42**, 5224–5242 (2005)
18. Zenkour A.M.: A comprehensive analysis of functionally graded sandwich plates: Part 2-Buckling and free vibration. *Int. J. Solids Struct.* **42**, 5243–5258 (2005)
19. Zenkour A.M.: Generalized shear deformation theory for bending analysis of functionally graded plates. *Appl. Math. Model.* **30**, 67–84 (2006)
20. Zenkour A.M., Alghamdi N.A.: Thermoelastic bending analysis of functionally graded sandwich plates. *J. Mater. Sci.* **43**, 2574–2589 (2008)
21. Kadoli R., Akhtar K., Ganesan N.: Static analysis of functionally graded beams using higher order shear deformation theory. *Appl. Math. Modell.* **32**, 2509–2525 (2008)
22. Reddy J.N.: Analysis of functionally graded plates. *Int. J. Numer. Meth. Eng.* **47**, 663–684 (2000)
23. Reddy J.N., Chin C.D.: Thermomechanical analysis of functionally graded cylinders and plates. *J. Thermal Stresses* **21**, 593–626 (1998)
24. Reddy J.N., Cheng Z.Q.: Three-dimensional thermomechanical deformations of functionally graded rectangular plates. *Eur. J. Mech. A Solids* **20**, 841–855 (2001)
25. Arciniega R.A., Reddy J.N.: Large deformation analysis of functionally graded shells. *Int. J. Solids Struct.* **44**, 2036–2052 (2007)
26. Praveen G.N., Reddy J.N.: Nonlinear transient thermoelastic analysis of functionally graded ceramic–metal plates. *Int. J. Solids Struct.* **35**, 4457–4476 (1998)

27. Chakraborty A., Gopalakrishnan S., Reddy J.N.: A new beam finite element for the analysis of functionally graded materials. *Int. J. Mech. Sci.* **45**, 519–539 (2003)
28. Jin Z.H., Paulino G.H.: A viscoelastic functionally graded strip containing a crack subjected to in-plane loading. *Eng. Fract. Mech.* **69**, 1769–1790 (2002) Ghosh M.K., Kanoria M.: Analysis of thermoelastic response in a functionally graded spherically isotropic hollow sphere based on Green–Lindsay theory. *Acta Mech.* **207**, 51–67 (2009)
29. Ueda S.: A cracked functionally graded piezoelectric material strip under transient thermal loading. *Acta Mech.* **199**, 53–70 (2008)
30. Li X.Y., Ding H.J., Chen W.Q.: Axisymmetric elasticity solutions for a uniformly loaded annular plate of transversely isotropic functionally graded materials. *Acta Mech.* **196**, 139–159 (2008)
31. Etemadi, E., Khatibi, A.A., Takaffoli, M.: 3D finite element simulation of sandwich panels with a functionally graded core subjected to low velocity impact. *Compos. Struct.* (2009) (in press)
32. Anderson T.A.: A 3-D elasticity solution for a sandwich composite with functionally graded core subjected to transverse loading by a rigid sphere. *Compos. Struct.* **60**, 265–274 (2003)
33. Ávila A.F.: Failure mode investigation of sandwich beams with functionally graded core. *Compos. Struct.* **81**, 323–330 (2007)
34. Bhangale R.K., Ganesan N.: Thermoelastic buckling and vibration behavior of functionally graded sandwich beam with constrained viscoelastic core. *J. Sound Vib.* **295**, 294–316 (2006)
35. Pradhan S.C., Murmu T.: Thermo-mechanical vibration of FGM sandwich beam under variable elastic foundations using differential quadrature method. *J. Sound Vib.* **321**, 342–362 (2009)
36. Ying J., Lü C.F., Chen W.Q.: Two-dimensional elasticity solutions for functionally graded beams resting on elastic foundations. *Compos. Struct.* **84**, 209–219 (2008)
37. Aköz A.Y., Kadioğlu F.: The mixed finite element solution of circular beam on elastic foundation. *Comput. Struct.* **60**(4), 643–651 (1996)
38. Chen W.Q., Lü C.F., Bian Z.G.: A mixed method for bending and free vibration of beams resting on a Pasternak elastic foundation. *Appl. Math. Model.* **28**, 877–890 (2004)
39. Matsunaga H.: Vibration and buckling of deep beam-columns on two-parameter elastic foundations. *J. Sound Vib.* **228**, 359–376 (1999) Sato M., Kanie S., Mikami T.: Mathematical analogy of a beam on elastic supports as a beam on elastic foundation. *Appl. Math. Model.* **32**, 688–699 (2008)
40. Tsiatas, G.C.: Nonlinear analysis of non-uniform beams on nonlinear elastic foundation. *Acta Mech.* (2009) (in press)
41. Il'yushin, A.A., Pobedrya, B.E.: Foundation of mathematical theory of thermo viscoelasticity. Moscow: Nauka (1970) (in Russian)
42. Allam, M.N.M., Pobedrya, B.E.: On the solution of quasi-static problem in anisotropic viscoelasticity. *ISV Acad Nauk Ar SSR, Mech.* **31**, 19–27 (1978) (in Russian)
43. Zenkour A.M.: Buckling of fiber-reinforced viscoelastic composite plates using various plate theories. *J. Eng. Math.* **50**, 75–93 (2004)



# EFFECT OF TRANSVERSE NORMAL AND SHEAR DEFORMATION ON A FIBER-REINFORCED VISCOELASTIC BEAM RESTING ON TWO-PARAMETER ELASTIC FOUNDATIONS

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## Abstract:

This article investigates the effect of transverse normal and shear deformations on a fiber-reinforced viscoelastic beams resting on two-parameter (Pasternak's) elastic foundations. The results are obtained by the refined sinusoidal shear deformation beam theory and compared with those obtained by the simple sinusoidal shear deformation beam theory, Timoshenko first-order shear deformation beam theory as well as Euler-Bernoulli classical beam theory. The effects of foundation stiffness on bending of viscoelastic composite beam are presented. The effective moduli methods are used to derive the governing equations of viscoelastic beams. The influences of several parameters, such as length-to-depth ratio, foundation stiffness, time parameter and other parameters on mechanical behavior of composite beams resting on Pasternak's foundations are investigated. Numerical results are presented and conclusions are formulated.

**KeyWords:** Pasternak's foundations; Viscoelastic composite beam; Transverse normal and shear deformation; Refined sinusoidal beam theory

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## References:

- A. Y. Aköz and F. Kadioğlu, *Comput. Struct.* **60**(4), 643 (1996).
- M. N. M. Allam and P. G. Appleby, *Res. Mech.* **19**, 113 (1986).
- M. N. M. Allam and B. E. Pobedrya, *ISV Acad. Nauk Ar SSR, Mech.* **31**, 19 (1978).
- M. N. M. Allam and A. M. Zenkour, *Appl. Math. Model.* **27**, 233 (2003), DOI: 10.1016/S0307-904X(02)00123-3 .
- E. Carrera, *Arch. Comput. Meth. Engng.* **9**(2), 87 (2002), DOI: 10.1007/BF02736649 .
- E. Carrera and A. Ciuffreda, *Compos. Struct.* **69**, 271 (2005), DOI: 10.1016/j.compstruct.2004.07.003 .
- W. Q. Chen, C. F. Lü and Z. G. Bian, *Appl. Math. Model.* **28**, 877 (2004), DOI: 10.1016/j.apm.2004.04.001 .
- M. E. Ergüven and A. Gedikli, *Comput. Mech.* **31**, 229 (2003).
- D. V. Georgievskii, *Russian J. Math. Phys.* **14**(3), 262 (2007), DOI: 10.1134/S106192080703003X .
- Il'yushin, A. A. and Pobedrya, B. E. [1970] "Foundation of mathematical theory of thermo viscoelasticity," Moscow: Nauka [in Russian].

